

THE FULL SPIN STRUCTURE OF QUARKS IN THE NUCLEON¹

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Abstract

We discuss bounds on the distribution and fragmentation functions that appear at leading order in deep inelastic 1-particle inclusive leptonproduction or in Drell-Yan processes. These bounds simply follow from positivity of the quark-hadron scattering matrix elements and are an important guide in estimating the magnitude of the azimuthal and spin asymmetries in these processes. We focus on an example relevant for deep inelastic scattering at relatively low energies.

1 The spin structure in inclusive DIS

In deep-inelastic scattering (DIS) the transition from hadrons to quarks and gluons is described in terms of distribution and fragmentation functions. In general, the distribution functions for a quark can be obtained from the lightcone correlation functions [1, 2, 3, 4].

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \psi_i(\xi) | P, S \rangle \Big|_{\xi^+ = \xi_T = 0}, \quad (1)$$

depending on the lightcone fraction $x = p^+/P^+$. The hadron momentum P is chosen so that it has no transverse component, $P_T = 0$. At leading order, the relevant part of the correlator is $\Phi\gamma^+$

$$(\Phi\gamma^+)_{ij} = \int \frac{d\xi^-}{2\pi\sqrt{2}} e^{ip \cdot \xi} \langle P, s' | \psi_{+j}^\dagger(0) \psi_{+i}(\xi) | P, s \rangle \Big|_{\xi^+ = \xi_T = 0} \quad (2)$$

where $\psi_+ \equiv P_+ \psi = \frac{1}{2} \gamma^- \gamma^+ \psi$ is the good component of the quark field [5].

The correlator contains all the soft parts appearing in the scattering processes and, as shown in Fig. 1, is related to the forward amplitude for antiquark-hadron scattering. By considering the quantity $M = (\Phi\gamma^+)^T$, one finds that for any antiquark-hadron state $|a\rangle$ the expectation value $\langle a | M | a \rangle$ must be larger than or equal to zero.

Thus our strategy is the following: express the forward scattering matrix M as a matrix in the [parton chirality space \otimes hadron spin space] and obtain our bounds by requiring it to be positive semi-definite.

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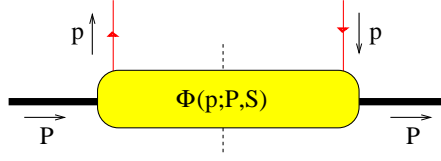


Figure 1: Matrix element for distribution functions.

At leading twist and when no partonic intrinsic transverse momentum is taken into account, $\Phi(x)\gamma^+$ is simply given by the contribution of three distribution functions [6]

$$\Phi(x)\gamma^+ = \left\{ f_1(x) + \lambda g_1(x) \gamma_5 + h_1(x) \gamma_5 \not{S}_T \right\} P_+. \quad (3)$$

The first step consists in writing this quantity as a matrix in the *parton* chirality space; this is easily done by using the explicit expression of $P_+ = \frac{1}{2}\gamma^-\gamma^+$ and γ_5 as 4×4 Dirac matrices. In chiral representation we find

$$M_{ij} = \begin{pmatrix} f_1(x) + \lambda g_1(x) & 0 & 0 & (S_T^1 + i S_T^2) h_1(x) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (S_T^1 - i S_T^2) h_1(x) & 0 & 0 & f_1(x) - \lambda g_1(x) \end{pmatrix} \quad (4)$$

As it is clear from the above expression, at leading twist there are only two relevant basis states, corresponding to (good components of) the right and the left handed partons. Thus, instead of using the full four dimensional Dirac space, we can effectively use only a two-dimensional chirality space.

Step two will be to write the above matrix explicitly in the *hadron* spin space. In order to study the correlation function in a spin 1/2 target we introduce a spin vector S that parameterizes the spin density matrix

$$\rho(P, S) = \frac{1}{2}(1 + \mathbf{S} \cdot \boldsymbol{\sigma}). \quad (5)$$

The spin vector satisfies $P \cdot S = 0$ and $S^2 = -1$ (spacelike) for a pure state, $-1 < S^2 \leq 0$ for a mixed state. Using $\lambda \equiv M S^+ / P^+$ and the transverse spin vector S_T , the condition becomes $\lambda^2 + \mathbf{S}_T^2 \leq 1$, as can be seen from the rest-frame expression $S = (0, \mathbf{S}_T, \lambda)$. The precise equivalence of a 2×2 matrix $\tilde{M}_{ss'}$ in the target spin space and the S -dependent function $M(S)$ is $M(S) = \text{Tr} [\rho(S) \tilde{M}]$. Explicitly, the S -dependent function

$$M(S) = M_O + \lambda M_L + S_T^1 M_T^1 + S_T^2 M_T^2, \quad (6)$$

corresponds to a matrix, which in the target rest-frame with as basis the spin 1/2 states with $\lambda = +1$ and $\lambda = -1$ becomes

$$\tilde{M}_{ss'} = \begin{pmatrix} M_O + M_L & M_T^1 - i M_T^2 \\ M_T^1 + i M_T^2 & M_O - M_L \end{pmatrix} \quad (7)$$

Thus, each element of matrix (4) will transform in a 2×2 matrix, according to Eqs.(6) and (7).

At leading twist and in absence of intrinsic transverse momentum, in the combined [parton chirality \otimes hadron spin space], the final result is

$$\tilde{M}_{is,js'} = \begin{pmatrix} f_1 + g_1 & 0 & 0 & 2h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix}. \quad (8)$$

From the positivity of the diagonal elements one recovers the trivial bounds $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$, but requiring the eigenvalues of the matrix to be positive gives the stricter Soffer bound [7],

$$|h_1(x)| \leq \frac{1}{2} (f_1(x) + g_1(x)). \quad (9)$$

2 The full spin structure in SIDIS

We now turn to the more general case in which non-collinear configurations are taken into account. Transverse momenta of the partons inside the proton play an important role in hard processes with more than one hadron [8], like semi-inclusive deep inelastic scattering (SIDIS), $e^- H \rightarrow e^- hX$ [9], or Drell-Yan scattering, $H_1 H_2 \rightarrow \mu^+ \mu^- X$ [10].

Analogous bounds can be obtained for transverse momentum dependent distribution and fragmentation functions. The soft parts involving the distribution functions are contained in the lightfront correlation function

$$\Phi_{ij}(x, \mathbf{p}_T) = \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}_j(0) \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0}, \quad (10)$$

depending on $x = p^+/P^+$ and the quark transverse momentum \mathbf{p}_T in a target with $P_T = 0$.

Separating the terms corresponding to unpolarized (O), longitudinally polarized (L) and transversely polarized targets (T), the most general parameterizations with p_T -dependence, relevant at leading order, are

$$\begin{aligned} \Phi_O(x, \mathbf{p}_T) \gamma^+ &= \left\{ f_1(x, \mathbf{p}_T^2) + i h_1^\perp(x, \mathbf{p}_T^2) \frac{\not{p}_T}{M} \right\} P_+, \\ \Phi_L(x, \mathbf{p}_T) \gamma^+ &= \left\{ \lambda g_{1L}(x, \mathbf{p}_T^2) \gamma_5 + \lambda h_{1L}^\perp(x, \mathbf{p}_T^2) \gamma_5 \frac{\not{p}_T}{M} \right\} P_+, \\ \Phi_T(x, \mathbf{p}_T) \gamma^+ &= \left\{ f_{1T}^\perp(x, \mathbf{p}_T^2) \frac{\epsilon_{T\rho\sigma} p_T^\rho S_T^\sigma}{M} + g_{1T}(x, \mathbf{p}_T^2) \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} \gamma_5 \right. \\ &\quad \left. + h_{1T}(x, \mathbf{p}_T^2) \gamma_5 \not{S}_T + h_{1T}^\perp(x, \mathbf{p}_T^2) \frac{\mathbf{p}_T \cdot \mathbf{S}_T}{M} \frac{\gamma_5 \not{p}_T}{M} \right\} P_+, \end{aligned} \quad (11)$$

where indeed $\Phi(x, \mathbf{p}_T) = \Phi_O(x, \mathbf{p}_T) + \Phi_L(x, \mathbf{p}_T) + \Phi_T(x, \mathbf{p}_T)$. As before, f_{\dots} , g_{\dots} and h_{\dots} indicate unpolarized, chirality and transverse spin distributions. The subscripts L and T indicate the target polarization, and the superscript \perp signals explicit presence of transverse momentum of partons. Using the notation $f^{(1)}(x, \mathbf{p}_T^2) \equiv (|\mathbf{p}_T|^2/2M^2) f(x, \mathbf{p}_T^2)$, one sees that $f_1(x, \mathbf{p}_T^2)$, $g_1(x, \mathbf{p}_T^2) = g_{1L}(x, \mathbf{p}_T^2)$ and $h_1(x, \mathbf{p}_T^2) = h_{1T}(x, \mathbf{p}_T^2) + h_{1T}^{\perp(1)}(x, \mathbf{p}_T^2)$ are the functions surviving p_T -integration.

To put bounds on the transverse momentum dependent functions, we again make the matrix structure explicit, following the same procedure we used in the previous simpler case in which no p_T was taken into account. We find for $M = (\Phi(x, \mathbf{p}_T) \gamma^+)^T$ the full spin matrix (for simplicity we do not explicitly indicate the x and \mathbf{p}_T^2 dependence of the distribution functions)

$$\tilde{M} = \begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi} g_{1T} & \frac{|p_T|}{M} e^{-i\phi} h_{1L}^\perp & 2(h_{1T} + h_{1T}^{\perp(1)}) \\ \frac{|p_T|}{M} e^{-i\phi} g_{1T}^* & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^\perp & -\frac{|p_T|}{M} e^{-i\phi} h_{1T}^{\perp*} \\ \frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp*} & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^\perp & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi} g_{1T}^* \\ 2(h_{1T} + h_{1T}^{\perp(1)}) & -\frac{|p_T|}{M} e^{i\phi} h_{1L}^\perp & -\frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 + g_{1L} \end{pmatrix}$$

where ϕ is the azimuthal angle of the transverse momentum vector. Here, we have left out the T-odd functions. But time-reversal invariance was not imposed in the parameterization of $(\Phi(x, \mathbf{p}_T) \gamma^+)$ in Eqs. (11), allowing for non-vanishing T-odd functions $f_{1T}^\perp(x, \mathbf{p}_T^2)$ and $h_{1T}^\perp(x, \mathbf{p}_T^2)$. They can be easily incorporated as the imaginary parts of the functions $g_{1T}(x, \mathbf{p}_T^2)$ and $h_{1L}^\perp(x, \mathbf{p}_T^2)$, to be precise $g_{1T} \rightarrow g_{1T} + i f_{1T}^\perp$ and $h_{1L}^\perp \rightarrow h_{1L}^\perp + i h_{1T}^\perp$. Possible sources of T-odd effects in the initial state have been discussed in Refs [11].

This matrix is particularly relevant, as it illustrates the full quark spin structure accessible in a polarized nucleon [12], which is equivalent to the full spin structure of the forward antiquark-nucleon scattering amplitude. Bounds to insure positivity of any matrix element can be obtained by looking at the 1-dimensional and 2-dimensional subspaces and at the eigenvalues of the full matrix. The 1-dimensional subspaces give the trivial bounds

$$f_1(x, \mathbf{p}_T^2) \geq 0, \quad (12)$$

$$|g_{1L}(x, \mathbf{p}_T^2)| \leq f_1(x, \mathbf{p}_T^2). \quad (13)$$

From the 2-dimensional subspaces we get

$$|h_1| \leq \frac{1}{2} (f_1 + g_{1L}) \leq f_1, \quad (14)$$

$$|h_{1T}^{\perp(1)}| \leq \frac{1}{2} (f_1 - g_{1L}) \leq f_1, \quad (15)$$

$$|g_{1T}^{(1)}|^2 \leq \frac{\mathbf{p}_T^2}{4M^2} (f_1 + g_{1L}) (f_1 - g_{1L}) \leq \frac{\mathbf{p}_T^2}{4M^2} f_1^2, \quad (16)$$

$$|h_{1L}^{\perp(1)}|^2 \leq \frac{\mathbf{p}_T^2}{4M^2} (f_1 + g_{1L}) (f_1 - g_{1L}) \leq \frac{\mathbf{p}_T^2}{4M^2} f_1^2, \quad (17)$$

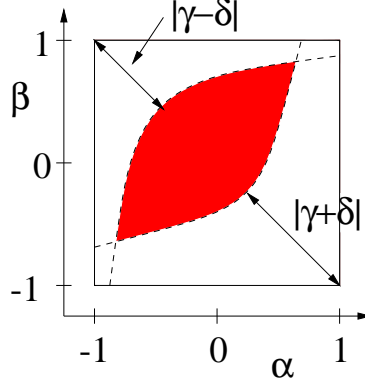


Figure 2: Allowed region (shaded) for α and β depending on γ and δ .

where, once again, we did not explicitly indicate the x and \mathbf{p}_T^2 dependence to avoid too heavy a notation. Besides the Soffer bound, Eq. (14), new bounds for the distribution functions are found. In particular, one sees that functions like $g_{1T}^{(1)}(x, \mathbf{p}_T^2)$ and $h_{1L}^{\perp(1)}(x, \mathbf{p}_T^2)$ appearing in azimuthal asymmetries in leptonproduction are proportional to $|\mathbf{p}_T|$ for small p_T .

Before sharpening these bounds via the eigenvalues, it is convenient to introduce two positive definite functions $A(x, \mathbf{p}_T^2)$ and $B(x, \mathbf{p}_T^2)$ such that $f_1 = A + B$ and $g_1 = A - B$ and define

$$h_1(x, \mathbf{p}_T^2) = \alpha A, \quad (18)$$

$$h_{1T}^{\perp(1)}(x, \mathbf{p}_T^2) = \beta B, \quad (19)$$

$$g_{1T}^{(1)}(x, \mathbf{p}_T^2) = \gamma \frac{|p_T|}{M} \sqrt{A B}, \quad (20)$$

$$h_{1L}^{\perp(1)}(x, \mathbf{p}_T^2) = \delta \frac{|p_T|}{M} \sqrt{A B}, \quad (21)$$

where the functions $\alpha(x, \mathbf{p}_T^2)$, $\beta(x, \mathbf{p}_T^2)$, $\gamma(x, \mathbf{p}_T^2)$ and $\delta(x, \mathbf{p}_T^2)$ have absolute values in the interval $[-1, 1]$. Note that α and β are real-valued but γ and δ are complex-valued, the imaginary part determining the strength of the T-odd functions.

Next we sharpen these bounds using the eigenvalues of the matrix, which are given by

$$e_{1,2} = (1 - \alpha)A + (1 + \beta)B \pm \sqrt{4AB|\gamma + \delta|^2 + ((1 - \alpha)A - (1 + \beta)B)^2}, \quad (22)$$

$$e_{3,4} = (1 + \alpha)A + (1 - \beta)B \pm \sqrt{4AB|\gamma - \delta|^2 + ((1 + \alpha)A - (1 - \beta)B)^2}. \quad (23)$$

Requiring them to be positive can be converted into the conditions

$$A + B \geq 0. \quad (24)$$

$$|\alpha A - \beta B| \leq A + B, \quad \text{i.e. } |h_{1T}(x, \mathbf{p}_T^2)| \leq f_1(x, \mathbf{p}_T^2) \quad (25)$$

$$|\gamma + \delta|^2 \leq (1 - \alpha)(1 + \beta), \quad (26)$$

$$|\gamma - \delta|^2 \leq (1 + \alpha)(1 - \beta). \quad (27)$$

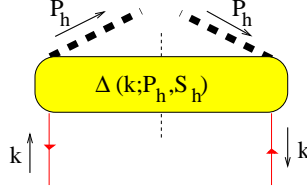


Figure 3: Matrix element for fragmentation functions.

It is interesting for the phenomenology of deep inelastic processes that a bound for the transverse spin distribution h_1 is provided not only by the inclusively measured functions f_1 and g_1 , but also by the functions $g_{1T}(x, \mathbf{p}_T^2)$ and $h_{1L}^\perp(x, \mathbf{p}_T^2)$, responsible for specific azimuthal asymmetries [9, 13]. This is illustrated in Fig. 2.

A perfectly analogous calculation can be performed for fragmentation functions, which describe the hadronization process of a parton into the final detected hadron. In this case the transverse momentum dependent correlator is [14]

$$\Delta_{ij}(z, \mathbf{k}_T) = \sum_X \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \psi_i(\xi) | P_h, X \rangle \langle P_h, X | \bar{\psi}_j(0) | 0 \rangle \Big|_{\xi^+=0}, \quad (28)$$

(see Fig. 3) depending on $z = P_h^+/k^+$ and the quark transverse momentum k_T leading to a hadron with $P_{hT} = 0$. A simple boost shows that this is equivalent to a quark producing a hadron with transverse momentum $P_{h\perp} = -z k_T$ with respect to the quark.

Like Φ , Δ is parameterized in terms of unpolarized, chirality and transverse-spin fragmentation functions [9], denoted by capital letters D_{\dots} , G_{\dots} , and H_{\dots} , respectively. For the fragmentation process, time-reversal invariance cannot be imposed [15, 16, 17], and the T-odd fragmentation functions D_{1T}^\perp [9] and H_1^\perp [18] play a crucial role in some azimuthal spin asymmetries as we shall discuss later on.

All the bounds obtained for the distribution functions can be rephrased in terms of the corresponding fragmentation functions. For instance, the relevant bounds for the Collins function $H_1^{\perp(1)}$, describing the fragmentation of a transversely polarized quark into a (spin zero) pion becomes

$$H_1^{\perp(1)}(z_\pi, \mathbf{P}_{\pi\perp}^2) \leq \frac{|\mathbf{P}_{\pi\perp}|}{2z_\pi M_\pi} D_1(z_\pi, \mathbf{P}_{\pi\perp}^2), \quad (29)$$

while for the other T-odd function $D_{1T}^{\perp(1)}$, describing fragmentation of an unpolarized quark into a polarized hadron such as a Λ one has

$$D_{1T}^{\perp(1)}(z_\Lambda, \mathbf{P}_{\Lambda\perp}^2) \leq \frac{|\mathbf{P}_{\Lambda\perp}|}{2z_\Lambda M_\Lambda} D_1(z, \mathbf{P}_{\Lambda\perp}^2). \quad (30)$$

Similarly to what happened for the distribution functions, a bound for the transverse spin fragmentation H_1^\perp is provided not only by the inclusive function D_1 but, when we sharpen the bounds by requiring positivity of the eigenvalues of the full matrix, the magnitude of H_1^\perp also constrains the magnitude of H_1 [19].

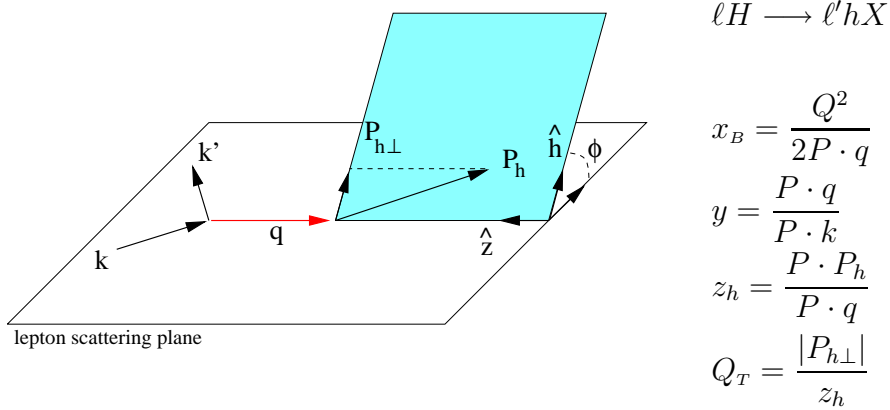


Figure 4: Kinematics for 1-particle inclusive lepton production.

Recently SMC [20], HERMES [21] and LEP [22] have reported preliminary results for azimuthal asymmetries. More results are likely to come in the next few years from HERMES, RHIC and COMPASS experiments. Although much theoretical work is needed, for instance on factorization, scheme ambiguities and the stability of the bounds under evolution [23], these future experiments may provide us with the knowledge of the full helicity structure of quarks in a nucleon. The elementary bounds derived in this paper can serve as important guidance to estimate the magnitudes of asymmetries expected in the various processes.

3 An example, relevant for JLAB@12 GeV

The asymmetries for which evidence recently has been found are mostly single spin asymmetries involving T-odd fragmentation functions, such as the Collins function H_1^\perp . We would like to discuss here a measurement that can be combined with the measurement of the inclusive structure function $g_2(x)$ in deep inelastic scattering off a transversely polarized target at large x .

The relevant kinematic variables are illustrated in Fig. 4, where also the scaling variables are introduced. Most often one considers the cross section integrated over all transverse momenta. But we emphasize that in principle the cross section can depend on a transverse vector, for which we use $\mathbf{q}_T = -P_{h\perp}/z_h$. This vector either represents the transverse momentum of the photon momentum q (with respect to the two hadrons, target and produced hadron) or the transverse momentum of the produced hadron (with respect to the target and photon momenta). Introducing the weighted cross sections

$$\langle W \rangle_{P_e P_H P_h} \equiv \int d\phi^\ell d^2 \mathbf{q}_T W(Q_T, \phi_h^\ell, \phi_S^\ell, \phi_{S_h}^\ell) \frac{d\sigma_{P_e P_H P_h}}{dx_B dy dz_h d\phi^\ell d^2 \mathbf{q}_T}, \quad (31)$$

where W is some weight depending on azimuthal angles and transverse momentum and the subscripts P_e , P_H and P_h are the polarizations of lepton, target and produced

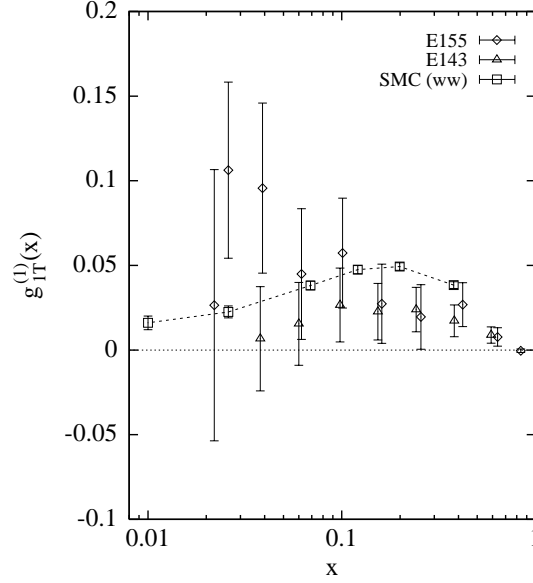


Figure 5: Estimate of $g_{1T}(x)$ obtained from g_2 data or from the Wandzura-Wilczek approximation using the SMC g_1 -data (see ref. [28] for details).

hadron respectively, we can construct several asymmetries.

To illustrate the weights, let's consider an easy example: the standard \mathbf{q}_T -integrated 1-particle inclusive unpolarized cross section,

$$\frac{d\sigma_{OO}}{dx_B dy dz_h} = \frac{2\pi\alpha^2 s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left(1 + (1-y)^2\right) x_B f_1^a(x_B) D_1^a(z_h), \quad (32)$$

in this language becomes

$$\langle 1 \rangle_{OO} = \frac{2\pi\alpha^2 s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left(1 + (1-y)^2\right) x_B f_1^a(x_B) D_1^a(z_h). \quad (33)$$

One of the leading asymmetries involving the function $g_{1T}(x, \mathbf{p}_T^2)$ discussed in the previous section is an asymmetry for longitudinally polarized leptons off a transversely polarized nucleon [24]

$$\left\langle \frac{Q_T}{M} \cos(\phi_h^\ell - \phi_S^\ell) \right\rangle_{LT} = \frac{2\pi\alpha^2 s}{Q^4} \lambda_e |\mathbf{S}_T| y(2-y) \sum_{a,\bar{a}} e_a^2 x_B g_{1T}^{(1)a}(x_B) D_1^a(z_h), \quad (34)$$

Since the fragmentation function involved is the standard leading one for unpolarized quarks into unpolarized or spin 0 hadrons, one can consider it for pion production, for which the fragmentation functions are reasonably well-known [25].

As we mentioned before, it is interesting to do this measurements together with the g_2 -measurement which, expressed as an (inclusive) asymmetry is given by

$$\langle \cos \phi_S^\ell \rangle_{LT} = -\lambda_e |\mathbf{S}_T| y \sqrt{1-y} \sum_{a,\bar{a}} e_a^2 \frac{M x_B^2}{Q} g_T^a(x_B) \quad (35)$$

where $g_T^a(x) = g_1^a(x) + g_2^a(x)$. The comparison of the inclusive measurement of g_2 and the semi-inclusive measurement of $g_{1T}^{(1)}$ would enable one to test the relation [26, 9, 27],

$$g_2^a(x) = \frac{d}{dx} g_{1T}^{(1)a}. \quad (36)$$

relating a twist three function to a transverse momentum dependent function. At present this relation can be used to estimate the function $g_{1T}(x)$ from existing g_2 measurements, of course in the same flavor averaged way as an inclusive measurement allows. The result is shown in Fig. 5, taken from ref. [28]. A measurement of the asymmetry in Eq. 34 allows an independent measurement of g_{1T} and a test of the above relation.

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